

EXERCICES DE RÉVISIONS : ANALYSE COMPLEXE-CHAPITRE I

Forme Cartésienne des Nombres Complexes

$$z = x + iy, \quad \bar{z} = x - iy.$$

$$x = \frac{z + \bar{z}}{2}, \quad y = \frac{z - \bar{z}}{2i}.$$

$$|z| = \sqrt{z\bar{z}} = \sqrt{x^2 + y^2} = r.$$

$$\arg(z) = \arctan \frac{y}{x}.$$

$$\frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i \frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}.$$

Forme Polaire des Nombres Complexes

$$z = r \cos \theta + ir \sin \theta = re^{i\theta}.$$

$$\bar{z} = r \cos \theta - ir \sin \theta = re^{-i\theta}.$$

$$|z| = \sqrt{z\bar{z}} = r.$$

$$\arg(z) = \theta = \arctan \frac{y}{x}.$$

$$\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}).$$

$$\sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta}).$$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}.$$

$$|z_1 z_2| = |z_1| |z_2| = r_1 r_2.$$

$$\arg(z_1 z_2) = \theta_1 + \theta_2.$$

$$\frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}.$$

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} = \frac{r_1}{r_2}.$$

$$\arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2.$$